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THE EXTRAGALACTIC DISTANCE SCALE

By Sidney van den Bergh

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Abstract

A Hubble constant $H = 120, +25, -20$ km./sec./mpc. is obtained from the luminosity classifications of members of the local group and a re-discussion of the distance modulus of the Andromeda galaxy.

Introduction. More than two hundred years ago the German philosopher Immanuel Kant speculated that many of the faint nebulous patches which astonomers had discovered in the sky might be extremely distant stellar systems similar to our own Milky Way. This hypothesis was fully confirmed by observations made during the first decades of the twentieth century. These observations showed that nebulae like those in Andromeda (M31) and Triangulum (M33) could be resolved into swarms of extremely faint stars. Subsequent studies revealed that these nebulae contained star clusters and types of variable stars which were already known in our own Milky Way system. The extreme faintness of these objects, compared to similar objects in our own galaxy, provided the first convincing proof for the great distances of the nebulae in which they were located.

Methods for determining the distances of galaxies. Distance determinations of extragalactic nebulae are based on the assumption that an object of a certain type, say a Cepheid variable with a period of 5 days, has the same intrinsic luminosity (absolute magnitude) in a distant galaxy as it has in our own Milky Way. Comparison of the presumably known *absolute* magnitude (M) of such a variable in our own galaxy with the *apparent* magnitude (m) of a similar variable in an extragalactic nebula then yields

the distance to that nebula (see appendix). In extragalactic research it is customary to express distances in terms of the distance modulus $(m - M)$. The distance modulus is equal to the apparent magnitude which a star of absolute magnitude zero would have. If for example the distance modulus of the Andromeda galaxy is 24, then a star with an absolute magnitude $M = 0$ would have an apparent magnitude $m = 24$. A very bright star with an absolute magnitude $M = -5$ would at this distance have an apparent magnitude $m = 19$. It will be clear from this example that, even in a relatively nearby galaxy like that in Andromeda, only the brightest objects can be used for distance determinations. In order of increasing brightness the objects which are most suitable for distance determinations are: (a) RR Lyrae type variables; (b) the brightest globular cluster stars; (c) Cepheid variables; (d) novae; (e) globular clusters; (f) the brightest spiral arm stars. These methods of distance determination can be used to determine the distances of the members of the small cluster of galaxies of which the Milky Way system is a member and which is usually referred to as the "local group". Beyond the local group only globular clusters and the brightest stars in the spiral arms of galaxies can be used as distance indicators. For galaxies with distance moduli larger than 32 even the brightest stars sink below the limit of detectability with the largest existing telescopes.

The expansion of the universe. In 1912 V. M. Slipher of the Lowell Observatory obtained the first radial velocity of an extragalactic nebula. Since that time many hundreds of radial velocities of galaxies have been measured, mainly by Humason at the Mount Wilson Observatory and Mayall at the Lick Observatory. These radial-velocity observations revealed the remarkable fact that almost all galaxies are receding from us, the velocity of recession being the greatest for the faintest and presumably most distant galaxies. An expansion of the universe as a whole is usually regarded as the most plausible explanation for this effect.

If one makes the reasonable assumption that the brightest nebulae in a cluster of galaxies have approximately the same absolute magnitude in all clusters, one finds that the velocity of recession of nebulae is directly proportional to the distance. This result may be expressed by the equation

$$V = HD \tag{1}$$

in which V is the velocity of recession, D is the distance and H is a constant of proportionality which is usually referred to as the Hubble constant. If H is known then the distance of any distant galaxy or cluster of galaxies may be determined from the observed velocity of recession.

Because the radial velocity of a galaxy can be determined with high precision, the accuracy with which large distances in the universe can be determined depends entirely on the accuracy of our knowledge of H. Unfortunately the determination of H is one of the most difficult problems in observational astronomy. The reason for this is the following: In addition to the velocity of recession due to the expansion of the universe each individual galaxy and each cluster of galaxies will have a random velocity which may be of the order of a few hundred kilometres per second. For those nearby galaxies in which Cepheids can be observed these random velocities are larger than the systematic velocity of recession due to the expansion of the universe. Equation (1) is therefore not applicable to nearby galaxies. Those galaxies to which equation (1) does apply and which are so distant that the systematic velocity of recession is much larger than their random velocity are too far away to observe Cepheids, novae or globular clusters in them.

The first determination of the Hubble constant. In 1936 Hubble obtained the value $H = 530$ km./sec./mpc. This value was obtained by using Shapley's (1930) period-luminosity relation for Cepheids to determine the distances of a number of nearby galaxies. With these distances Hubble then determined the absolute magnitudes of the brightest stars in a galaxy. Comparison of the apparent magnitudes of the brightest stars in the Virgo cluster with the absolute magnitudes determined in nearby galaxies then yielded the distance modulus and hence the distance of the Virgo cluster. From this distance and the known velocity of recession of the Virgo cluster the numerical value of the Hubble constant was derived using equation (1). This derivation is based on the assumption that the random velocity of the Virgo cluster as a whole is so small that it can be neglected. We now know that the value of H which Hubble obtained was much too large and that in consequence Hubble underestimated the distances of extragalactic nebulae considerably. Three factors were primarily responsible for Hubble's underestimate of the extragalactic distance scales.

1. The faint ends of the magnitude sequences used by Hubble in M31 and M33 were slightly in error. As a result his distance modulus for M31 should have been 22.4 rather than 22.0.

2. In deriving the distances of nearby galaxies Hubble used Shapley's (1930) period-luminosity relation for Cepheids. This period-luminosity relation was obtained by assuming the zero point of the period-luminosity relation (absolute magnitude of Cepheids with $\log P = 0$) to coincide with the luminosity of RR Lyrae stars. It is now realized that this period-

luminosity relation applies to the W Virginis type Cepheids of population II and not to the classical Cepheids of population I which Hubble observed in M31. Classical Cepheids are now believed to be about 1.5 magnitudes brighter than W Virginis stars of the same period.

3. The objects which Hubble regarded as the brightest stars in the Virgo cluster have been shown by Sandage (1958a) to be bright emission regions, similar to the Orion nebula, rather than individual stars. Such emission regions in which small star clusters are frequently embedded are considerably brighter than the brightest individual spiral arm stars.

The absolute magnitudes of Cepheids and RR Lyrae stars. The most accurate determinations of the absolute magnitudes of Cepheids are obtained for those Cepheids which are members of galactic clusters. The observational data which have been published so far are summarized in Table I. The table gives the absolute magnitude at median light $\bar{M}$ which is defined as

$$\bar{M} = \frac{M\,(\max) + M\,(\min)}{2} \tag{2}$$

and the logarithm of the period of pulsation.

TABLE I

ABSOLUTE MAGNITUDES OF CEPHEIDS IN CLUSTERS

Variable	M	Log P	Source
CEa Cas	−2.25	0.71	Sandage (1958b), Kukarkin *et al.* (1958)
CEb Cas	−2.20	0.65	Sandage (1958b), Kukarkin *et al.* (1958)
CF Cas	−2.60	0.69	Sandage (1958b)
DL Cas	−2.95	0.90	Arp, Sandage and Stephens (1959)
S Nor	−2.55	0.94	Irwin (1958), Arp (1958a)
U Sgr	−2.95	0.83	Eggen *et al.* (1957) Johnson (1957) Kron and Svolopoulis (1959)
EV Sct	−2.00	0.49	Arp (1958b)

From these data one obtains the period-luminosity relation (see appendix)

$$\bar{M} = -1.0 - 2.0 \log P \tag{3}$$

in which the slope of the period-luminosity relation has been obtained from Baade's (1956) investigation of the Cepheid variables in the Andromeda galaxy. The adopted value of the slope is quite uncertain and lies about half-way between the value obtained by Shapley (1940) and a provisional value obtained by Arp (1958c) in the Small Magellanic Cloud. Assuming the adopted slope to be correct the uncertainty of the zero point in equation (3) is about 0.1 magnitudes. A number of investigators including Arp, Kraft and Sandage are at present studying additional

Cepheids which may be members of galactic clusters. Their results, which should provide a more accurate calibration of the period-luminosity relation, are eagerly awaited.

From equation (3) one obtains a distance modulus $m - M \simeq 18.8$ for the Small Magellanic Cloud. This modulus is quite uncertain. It is based on the assumption that the period-luminosity relation in the Small Magellanic Cloud is identical to that in the galaxy. A number of recent investigations, in particular Arp (1959), van den Bergh (1958) and Kron and Svolopoulis (1959), suggest that young stars in the Small Magellanic Cloud may contain less heavy elements than those in the galaxy. If this should indeed prove to be the case, then the period-luminosity relations in these two systems might well be different. With the distance modulus of the Small Cloud given above and the observed median apparent magnitude $\bar{m} = 19.3$, of Small Cloud RR Lyrae stars, which was obtained by Thackeray and Wesselink (1955) (see appendix), one obtains $\bar{M} = +0.5$ for the RR Lyrae stars in the Small Magellanic Cloud. It should be emphasized that this value is quite uncertain. Recently Eggen and Sandage (1959) have presented evidence which also tends to indicate that the RR Lyrae stars are fainter than the usually adopted value $\bar{M} = 0.0$. It should however be kept in mind that the RR Lyrae stars probably do not form a homogeneous group and may have a considerable range in absolute luminosities (Notni 1957). Adopting $\bar{M} = +0.5$ rather than $\bar{M} = 0.0$ has the additional advantage that it greatly reduces the number of RR Lyrae stars, which on the basis of their proper motions, have hyperbolic velocities and could therefore escape from the galaxy. The adopted absolute magnitude of the RR Lyrae stars, together with the absorption towards the galactic centre derived by Kron and Mayall (1959), gives a more reasonable value for the distance to the galactic nucleus than is the case if $\bar{M} = 0.0$ is adopted. Baade's (1956) observation that the W Virginis variables in the Andromeda galaxy are about 1.5 magnitudes fainter than the classical Cepheids also tends to support $\bar{M} = +0.5$ for the RR Lyrae stars. The results obtained from Cepheids in galactic clusters, Cepheids and RR Lyrae stars in the Small Magellanic Cloud and Cepheids and W Virginis stars in M31 indicate that the *difference* between the zero points for Cepheids on the one hand and RR Lyrae and W Virginis stars on the other is approximately 1.5 magnitudes, the zero point of the Cepheids being at $\bar{M} \simeq -1.0$ that of the RR Lyrae and W Virginis stars being at $M \simeq +0.5$.

The distance of the Andromeda galaxy. The distance to the Andromeda galaxy (M31) can be regarded as the most fundamental yardstick for

measuring the extragalactic distance scale. The distance moduli of M31 which are obtained by different methods are discussed below.

(a) the brightest globular cluster stars:

According to Baade (1956) the brightest globular cluster stars in M31 have a visual magnitude of 21.2. Observations of the globular clusters in our own galaxy indicate that the RR Lyrae stars are about 3.0 magnitudes (visual) fainter than the brightest globular cluster stars. In M31 the RR Lyrae stars should therefore occur at visual magnitude 24.2. Assuming a colour index of 0.3 magnitudes and a reddening of 0.2 magnitudes by interstellar material between the sun and M31, one obtains $\bar{m} = 24.7$ for the RR Lyrae stars and hence a distance modulus $m - M = 24.2$.

(b) Cepheids:

The brightest Cepheid variables in the Andromeda galaxy obey the following period-luminosity relation

$$\bar{m} = 22.8 - 2.0 \log P \tag{4}$$

If one assumes that an intrinsic scatter exists about the mean period-luminosity relation, then the brightest Cepheids at a given period will be those which are intrinsically brightest and which also suffer negligible absorption. Sandage (1958) estimates that the difference between the brightest Cepheid at a given period and the mean magnitude of all Cepheids at that same period amounts to about 0.5 magnitudes. With this value one obtains the mean period-luminosity relation

$$\bar{m} = 23.3 - 2.0 \log P \tag{5}$$

From equations (3) and (5) a distance modulus $m - M = 24.3$ is obtained for the Andromeda galaxy.

(c) Novae:

From the relation between absolute magnitude at maximum light and the rate of fading of novae Schmidt (1957) obtains a distance modulus $m - M = 24.45 \pm 0.30$ for the Andromeda galaxy. (As in the case of the distance modulus derived from the Cepheids, this value refers to zero absorption within the Andromeda galaxy itself.)

(d) Globular Clusters:

Kron and Mayall (1959) have compared the apparent magnitudes of the brightest globular clusters in the Andromeda galaxy with the brightest globular clusters in the galaxy. Assuming $\bar{M} = +0.5$ for the RR Lyrae stars this comparison yields a distance modulus $m - M = 23.8$ for the Andromeda galaxy (see appendix).

The distance moduli obtained above are compared in Table II. It will be seen from the table that the agreement between the distance moduli

obtained by various methods is fairly good and certainly no worse than would be expected from the uncertainties inherent in each type of distance determination.

TABLE II

DISTANCE MODULI OF THE ANDROMEDA NEBULA

Method	m−M
Brightest globular cluster stars	24.2
Cepheid variables	24.3
Novae	24.45±0.3
Globular Clusters	23.8
mean	24.2

The mean value $m - M = 24.2$ will be adopted in the discussion of the distance scale beyond the local group.

Distances beyond the local group. Baum (1955) has determined the distance of the Virgo cluster by comparing the brightness of the globular clusters in the giant elliptical galaxy M87 with those in the Andromeda galaxy. This comparison indicates that the globular clusters in the Virgo cluster are 6.0 magnitudes fainter than those in the Andromeda galaxy yielding a provisional distance modulus $m - M = 30.2$ for the Virgo cluster. Sandage (1958a) has attempted to use the brightest star criterion to determine the distance of the Virgo cluster. His results are however somewhat inconclusive. If it could be shown that the intrinsic scatter in the luminosities of supernovae of a given type is small, they might perhaps be used to calibrate the extragalactic distance scale. However the data which are available at present are not yet sufficient to undertake such a calibration.

A different approach to the problem of the extragalactic distance scale beyond the local group is provided by considering entire galaxies as distance indicators. Such an approach has the advantage that it can be used to much greater distances than those methods which rely on the brightness of individual types of stars or globular clusters within a galaxy. Recently the author (1960) has presented evidence which appears to suggest that the absolute luminosity of late-type galaxies is strongly correlated with the degree to which their spiral structure is developed. The correlation is in the sense that the most luminous galaxies have the most strongly developed spiral structure. This effect has been used to establish a two-dimensional classification system for galaxies. The classification parameters are the Hubble *type* (Sa, Sb, Sc, Ir) and the luminosity *class* (I = supergiant galaxy, II = bright giant galaxy, III = normal

giant galaxy, IV = subgiant galaxy and V = dwarf galaxy). By means of this classification system it is possible to determine the absolute magnitude of a galaxy with an accuracy of 0.5 magnitudes.

From equation (1) the absolute magnitudes of distant galaxies of a given luminosity class can be determined in terms of the Hubble constant, if both their velocities of recession and apparent magnitudes are known. The absolute magnitudes of galaxies of the same luminosity class in the local group can however be determined directly from observations of variable stars, globular clusters, etc. The luminosity classification of galaxies in the local group therefore provides a means of determining the Hubble constant. The luminosity classifications of five members of the local group are given in Table III. The third column of the table gives the distance moduli of the local group members recently quoted by Sandage (1958a) but reduced by 0.4 magnitudes to bring them into agreement with the distance modulus of the Andromeda galaxy which has been obtained in the previous section. The last column gives the individual distance moduli derived from the luminosity classifications and a Hubble constant $H = 120$ km./sec./mpc.

TABLE III

DISTANCE MODULI OF LOCAL GROUP MEMBERS
($H = 120$ km./sec./mpc.)

Name of Galaxy	Type and Luminosity Class	m − M (Sandage) minus 0.4	m − M from Luminosity Class
M31	Sb I–II	24.2	23.7
M33	Sc II–III	24.1	24.5
LMC	Ir/SBc III–IV	18.8	18.2
SMC	Ir IV/IV–V	18.8	18.6
NGC 6822	Ir IV–V	23.7	24.6

It will be seen from the table that the two methods of determining the distance moduli of the members of the local group agree in the mean if a Hubble constant $H = 120$ km./sec./mpc. is adopted. The uncertainty of this result is estimated to be 0.4 magnitudes if the adopted distance moduli of the local group members are correct. The corresponding uncertainty in the Hubble constant is $H = 120, +25, -20$ km./sec./mpc.

Conclusion. The numerical value of H is not only important because it provides us with a means of converting the observed radial velocities of galaxies to distances but also because the Hubble constant is related to the age of the universe. The exact nature of this relationship depends on the type of universe in which we are living. The most recent determinations

of the ages of the oldest stars in the galaxy by Hoyle (1959) and Haselgrove and Hoyle (1959) yield values of about 14×10^9 years. For simple exploding world models this implies that $H \leqslant 50$ km./sec./mpc. which is incompatible with the value of the Hubble constant obtained above. Possibly this discrepancy indicates that we are not living in a simple exploding type of universe or alternately recent calculations may overestimate the age of the oldest stars in the galaxy.

I am indebted to Dr. G. de Vaucouleurs for making available to me prints of the Magellanic Clouds which were suitable for classification purposes and to Drs. Kron and Mayall for providing me with a copy of their symposium paper presented at the Toronto meetings of the American Astronomical Society.

Appendix

All magnitudes used in this paper have been reduced to the UBV system by the relation

$$P - V = 1.10\,(B - V) - 0.18 \qquad \text{(Allen 1955)}$$

To simplify the presentation of the data the symbol m is used for the apparent blue magnitude B and M is used for the absolute blue magnitude M_B.

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A COMPOUND INTERFEROMETER*

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Observations. Initial construction of the four-element interferometer was completed in the spring of 1958, and a receiver was put into operation at that time. Although subsequent observations were disturbed by various tests and alterations, periods of varying extent occurred when exceedingly good records were obtained. By the beginning of 1959, operations on the system of adjustments which have been described were complete, and practically continuous records of daily drift-curves exist from that time.

When several drift-curves are obtained with the successive use of narrower and narrower fan-shaped antenna patterns, sharp features are sometimes found in the scans obtained with the narrowest pattern. This is illustrated in the four drift-curves obtained on May 29, 1958 (see figure 6). The upper curve was obtained with the DC radiometer using the array with its 8 minutes of arc beam. The second drift-curve was obtained with the phase-sensitive radiometer using the array with one grating element, which gives a principal lobe of about 4.8 minutes of arc. The third curve was obtained with the array and the two-element grating, which gives a 2.4 minute of arc beam; while the bottom curve was obtained with the array and all four elements, which combination gives a beamwidth of about 1.2 minutes of arc. In this figure the horizontal bars and the connecting vertical lines represent the east-west extent of the photosphere, which is 31.5 minutes of arc. These four records were obtained within an interval of one-half hour, and it is known from patrol telescopes that the total flux remained constant during this period. The pointed feature on the left-hand side of the lowest drift-curve which was taken with the high resolution antenna is quite similar to the predicted antenna pattern, both in the appearance and half-width of the main lobe, and in the indication of negative side-lobes. The base for this drift-curve of the radio emissive region appears as a smooth round curve which may be regarded as a much broader region involving the same strip as the point source as well as adjacent ones. This point-like feature was one of the first to be observed and has been taken as one indication that the antenna is working as designed. In the series of drift-curves, the sharpening of this feature when scanned with successively sharper beams is obvious. The other prominent feature which appears to

*National Research Council Reprint No. 5509, concluded.

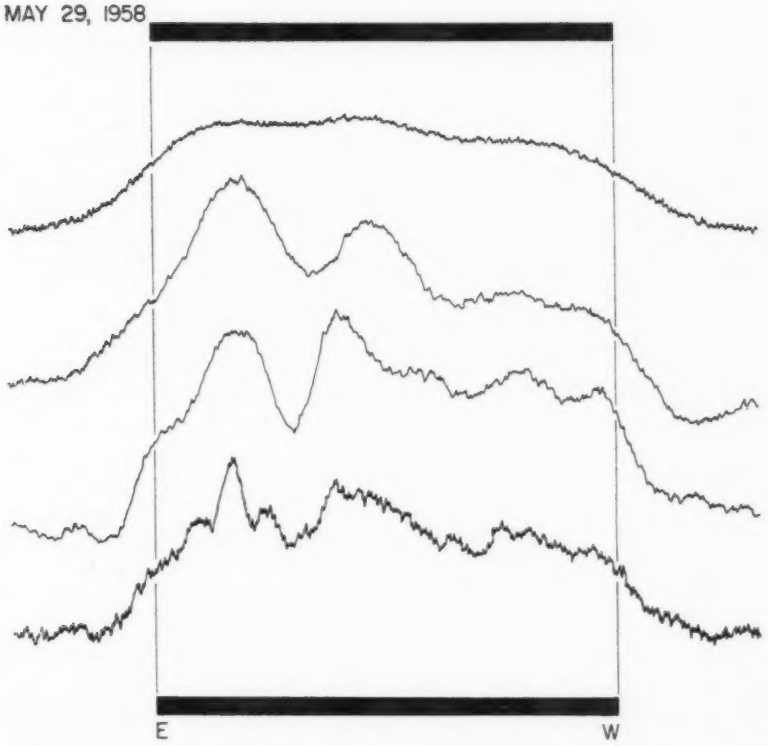

FIG. 6—Series of four drift curves taken on May 29th, 1958, and obtained with (a) the 150-foot array alone; (b) the 150-foot array and B_1; (c) the 150-foot array and B_1 and B_2; (d) the 150-foot array and B_1, B_2, B_3, and B_4.

the west, shows only a broadening, and indicates that this region or combination of radio-emissive regions has east-west dimensions several times that of the antenna pattern. The four curves shown here have not been normalized to have the same intensity scale, but are traced from original records.

In comparing the scanning curve taken with the 2.4 minute of arc beam with that taken with the 1.2 minute of arc beam, it will be noticed that the half-width of the point-like profile has been reduced by a factor of two. For the antenna patterns under consideration this is as it should be. The reduction in drift-curve half-width has been studied for 108 other

radio spots observed during a nine-month period, June 1958 to March 1959. These were selected from a total of 900 radio-emissive regions on the basis of their being single, isolated regions. The remaining radio spots showed pronounced broad features which may be related either to a true structure, or to the fortuitous coincidence of two or more spots in the same or adjacent strips, located in various combinations in the northern and southern hemispheres. Since a spot exists for several days on the solar disk, some of the spots appear more than once in this count. The distributions of widths of the 108 spot drift-curves obtained with the two compound interferometers (one with the two-element grating, and the other with the four-element grating) are shown in figure 7. Vertical lines

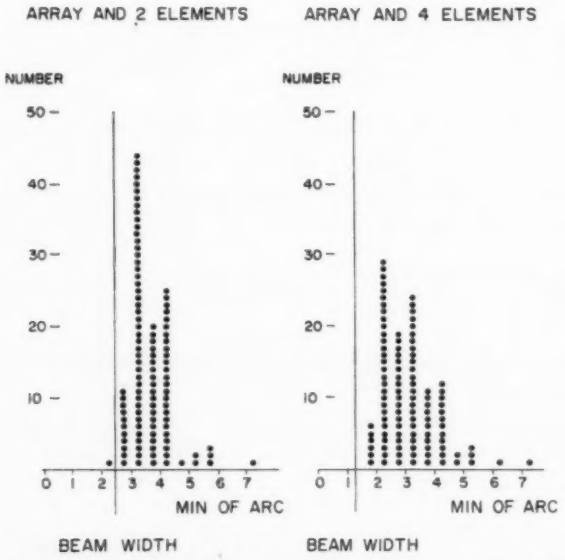

FIG. 7—Distribution of half-widths of the drift curves of the same isolated radio-emissive regions. Right-hand curve obtained with 600-foot compound interferometer; left-hand curve obtained with 300-foot compound interferometer.

have been placed at the half-widths of the two antenna patterns so that if a point source were scanned, the two dots representing the observations would lie on these lines. It is seen that none of the 108 spots are point sources but rather vary in their range of half-widths. For the 2.4 minute of arc beam, the majority of spots have half-widths lying between 3 and

4.5 minutes of arc, with several spots presenting widths close to the antenna beamwidth and several with widths several times greater. For the 1.2 minute of arc beam, the peak of the distribution is less pronounced with no spots close to the value of the antenna beamwidth and several with widths comparable to the broadest observed with the broader antenna pattern. The trend for the majority of the spots is to smaller half-widths when observed with the array and four elements, which indicates that the narrower antenna pattern is achieving the desired resolution.

The width of a spot drift-curve represents the combined effect of the natural width of the spot and the width of the antenna pattern. If the width of the most frequently occurring spot drift-curve for this period of time is assumed to be that of a typical spot, then in the case of the high resolution antenna, this observed width indicates that the typical source is a line source with a width of 1.5 minutes of arc (Covington and Harvey 1959), or a circular disk with a width of 1.8 minutes of arc. While a few of the spot-drift curves indicate sources which are narrower, the majority of those examined indicate broader sources and the investigation suggests that the east-west extent of the radio emissive regions associated with sun-spots is seldom smaller than 1.5 minutes of arc.

High resolution daily drift-curves taken about noon for two periods, January 2 to 6, 1959, and February 9 to 16, 1959, have been selected, and are shown in figures 8, 9(a), and 9(b). The first series shows the appearance on the east limb of a large sun-spot which is well isolated by January 6. At the beginning of this series the spot is blended with another, while on the remaining three days, definite separations may be seen. On January 5, an extra spot appears between the two large spots and is probably entirely fictitious, as such effects are known to arise from the interference of two prominent regions, situated in relation to each other in such a way that their superimposed side-lobes add. This enhanced side-lobe has a width approximately equal to that of the main lobe of the antenna pattern, and enables the broadening of the large spot profiles to be readily seen.

The second series, in February, shows the limb development of a more complicated region which, from the optical evidence, is most likely associated with an extensive east-west series of sun-spots in the northern hemisphere. On February 11, a radio spot on the east limb is evident as a broadening in the lower part of the pre-existing strong radio-emissive region, while by February 12 this same region has developed into a well isolated point-like spot. Subsequent daily drift-curves show the further development of these two regions. Although a small spot is apparent between the two large spots on February 14, and might be regarded as an

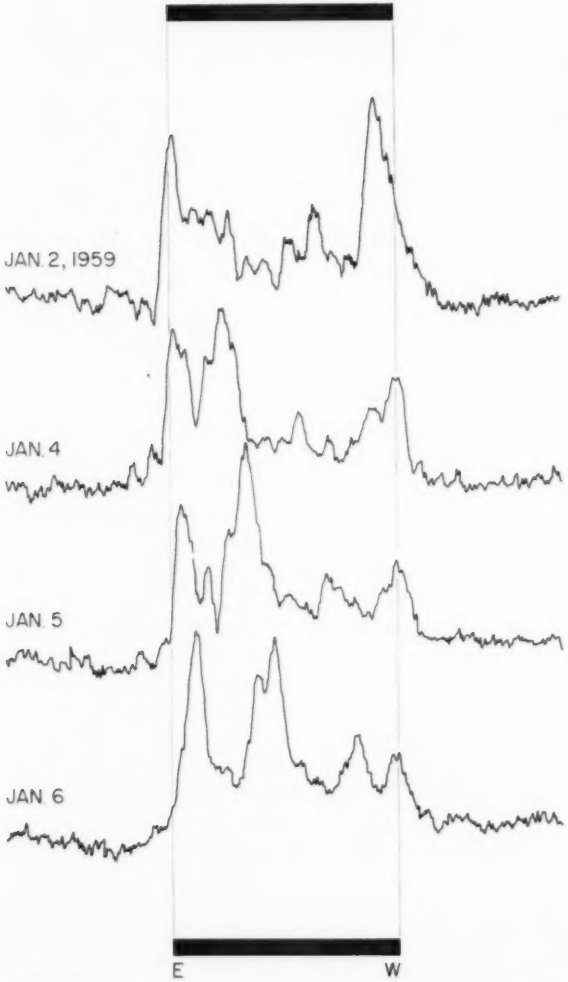

FIG. 8—Series of daily drift curves taken with 600-foot compound interferometer, January 2, 1959 to January 6, 1959.

enhanced side-lobe, its subsequent appearance on February 15 suggests that it is a real feature which was confused previously with the side-lobe. The last two series of drift curves which are displayed on rectangular co-

ordinates have been copied from the original records made on paper with curved co-ordinates. Although the curves were copied without normalization, the gain of the receiver remained approximately constant during each period so that approximate intercomparison may be made.

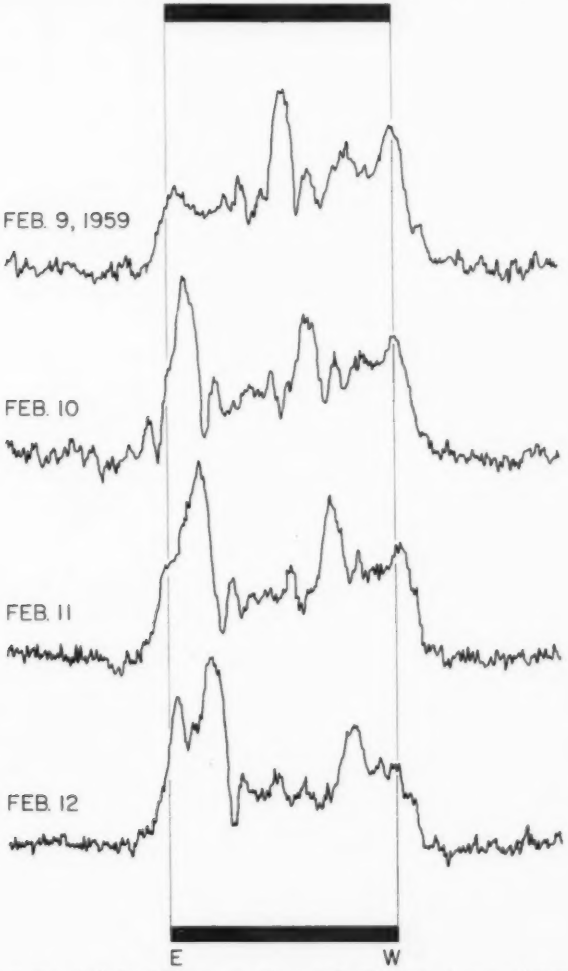

FIG. 9a—Series of daily drift curves taken with 600-foot compound interferometer, February 9 to February 12, 1959.

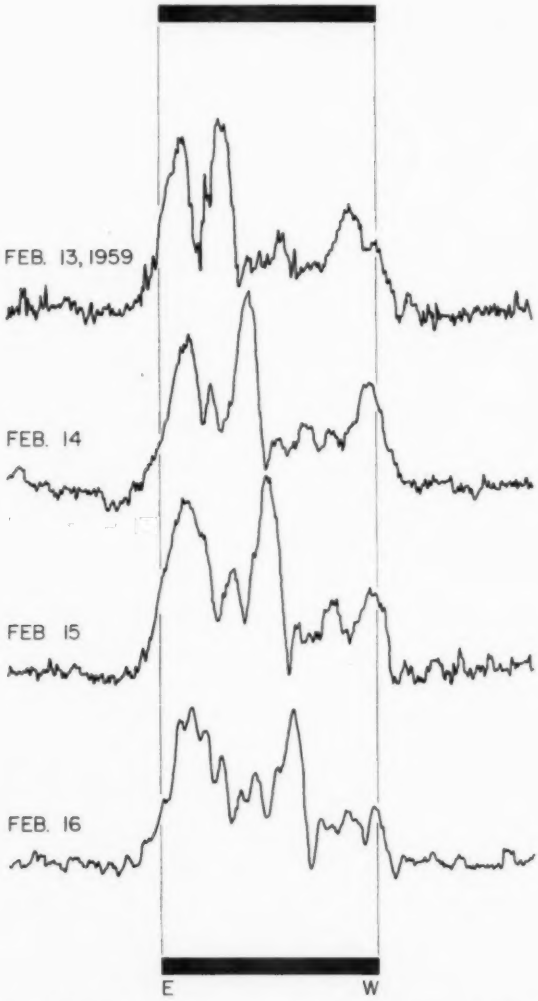

FIG. 9b—Series of daily drift curves, continuing from February 13 to February 16, 1959.

Stability of Squint Angle. The stability of the antenna pattern has been evaluated both qualitatively and quantitatively. In the first place, the

nature of the antenna is sufficiently complex that even a small instability gives rise to a marked deterioration of the quality of the drift curves and consequently a long series of good drift curves necessarily implies stable performance of the system. In the second place, it may be assumed that the geometrical centres of the radio and the optical emissions of the solar disk coincide and that it is reasonable to use the position of the optical feature to determine the squint angle of the radio beam. Likewise the assumptions—that the centre of the orbit of a radio-emissive region lies on the solar axis of rotation, and that the solar noise burst, at these wavelengths at least, coincides with the bright region of the flare—may be used to determine the position of the antenna pattern. The results of such investigations will be reviewed, and intercompared in order to evaluate the squint angle and its stability.

Daily values of the squint angle for the third drift curve, which makes use of the whole aperture, have been calculated for several months. Each value is determined by appropriate calculation from a knowledge of the time when the centre of the radio disk is in the antenna beam. The centre of the undisturbed solar radio disk, as determined by the mid-point of the elevated portion of the drift-curve upon which the radio spots appear, has been assumed to coincide with the centre of the photosphere. The day-to-day variations of the calculated squint angle were found to be about ± 2 min. of arc. Generally, these variations can be attributed to real changes in the centre of the radio disk as previously defined because of the predominance of emissive regions on one limb or the other, although in some cases the shift is fictitious due to the masking of the apparent quiet sun drift-curve by the side-lobe reception of strong radio-emissive regions just inside the limb. When monthly means are taken of the calculated squint angles, these variations are reduced to much smaller values. The monthly means for two successive winter months in 1958 were in close agreement and had an average value of $4°21'24''$; the monthly means for two successive summer months in 1959 were also in close agreement and had an average value of $4°22'48''$. The observed difference in squint angle of $1'24''$ does not introduce any serious degeneration of the resultant antenna pattern but nevertheless should be explained, even though at present there appears to be no clue. One possible cause may lie in the adjustments of the squint angle of the long array pattern which are required to compensate for a maximum difference in the array temperature between winter and summer of 50° C. As previously mentioned, the frequency of reception is altered slightly to change the squint angle in the opposite sense to that introduced by the temperature variations and, between seasons, a maximum angular shift of 5 minutes of arc must be

introduced. This is required to keep the maximum of the single lobe of the array centred upon the selected fixed lobe of the grating pattern.

The position of the radio-emissive regions may be observed from day to day and measured in terms of an angular distance from the centre of the radio disk. The positions of the strong, well isolated radio-emissive regions which exist from limb to limb lie about smooth curves theoretically determined by the angular position of the solar axis of rotation with respect to the fan-shaped beam, the latitude of the radio spot, and its elevation above the photosphere. The deviations about these smooth curves are most likely related to the reading errors and to daily variations of the squint angle. The observed departures from the smooth curves suggest that the stability of the squint is maintained for considerable periods of time to a relative accuracy of $\pm 1/8$ min. of arc. In fitting the theoretical curves to the observed curves, it has sometimes been found necessary to introduce a small correction to the radio determination of the centre of the solar disk. Geometrically, this is equivalent to the requirement that the centre of the orbit of the radio spot must lie upon the solar axis of rotation. For the trajectory of the very active region which appeared on the eastern limb June 9, 1959, a correction of 35 seconds of arc was found to be necessary and was in the sense to reduce the observed difference between the mean values of squint angle as observed in winter and in summer. However, it does not completely remove this discrepancy.

A well-defined and centrally-located small flare occurred at 16:36 U.T. on December 18, 1958, and a drift curve of the accompanying radio emission was obtained. The burst appeared as a small obvious enhancement on one drift-curve in the emission of a pre-existing region as recognized from comparison with other drift-curves, and the radio strip position of the burst was found to be 15 sec. of arc west of the flare region. This location involves a knowledge of the centre of the radio disk which was found by using the monthly mean squint as previously discussed. The half-width of the burst drift-curve was close to the half-width of the antenna pattern and indicates that a point-like feature was being observed. The discrepancy in optical and radio positions of the flare region is of the same magnitude as the observed scatter in the daily positions of the radio-emissive regions. It is about 1/8 of the beamwidth and has been taken as setting the relative limit with which the squint angle may be determined. However, the unexplained seasonal variation in the determination of the squint angle is much larger than this, amounting to a value of about 1 min. of arc, and thus sets the limit to the true value of the squint as presently known.

Conclusions. The operation of a large 10 cm. interferometer which forms a single-lobed, fan-shaped antenna pattern, 2° N-S by 1.2 min. of arc E-W has been described. The narrow profile of the power pattern approximates the functional form sin x/x where x is the East-West angular variable. This pattern, in comparison to the antenna pattern which is derived by collecting energy uniformly from the same linear aperture, has been shown to yield an increase in resolution of 1.3 times. In addition, the drift-curves of the sun obtained with this interferometer are automatically the Fourier series approximation with the highest spatial frequency given by the overall length of the antenna (Covington and Harvey 1959). Drift curves obtained with other antenna patterns require modification to yield this type of approximation of the source.

The slit-like shape of the pattern which is characteristic of the present compound interferometer, does have the limitation of being unable to provide information about the fine structure of any single source in the North-South direction as well as being unable to separate two or more sources lying in the same North-South line. The latter situation can sometimes be detected by observing the day-to-day positions of spots on the solar disk as the sun rotates, and will be completely absent during periods of low sun-spot activity. The first disadvantage has been accepted in view of the novel features of this type of antenna which produce narrow antenna patterns for scanning objects in one dimension only. In this case, the limitation in the accuracy with which the antenna pattern is known will be the most serious limitation in the newly obtained information. In other designs of this type of instrument, provision for reducing the North-South pattern to an extent comparable to the East-West pattern may be feasible by the use of elements which outline a square.

The present interferometer performance has been tested by taking daily observations of the sun and the results appear to be satisfactory. A minimum broadening of the drift curves of the radio-emissive regions appears and may be interpreted as a minimum dimension in the East-West direction of these radio-emissive regions, as 1.5 min. of arc if a linear E-W model is assumed, or as 1.8 min. of arc if a circular model is assumed.

Acknowledgments. The author is pleased to acknowledge the assistance given by the personnel of the Radio and Electrical Engineering Division of the National Research Council. In particular, Mr. W. A. G. Kennedy has supervised the mechanical construction of the antenna and Mrs. G. A. Harvey has performed various calculations in the reduction of the data.

CORRECTION:

On page 20 (this JOURNAL, vol. 54), the magnitude of the equal and opposite phase changes between the two different antennas A and B in reference to their mid-point should be increased by a factor of 2 to read $\pi(D/\lambda)\theta$ and equation (1) should be written

$$V(\theta) = k\left[A(\theta) \exp - j\left(\pi \frac{D}{\lambda}\theta\right) + B(\theta) \exp j\left(\pi \frac{D}{\lambda}\theta + \alpha t\right)\right] \times$$

$$\times \exp j(\omega t + \phi_r)$$

This correction carries through the development with the resulting appearance in the argument of the cosine term in equation (3) of a factor of 2.

However, in the application of equation (3) to describe the operation of the compound interferometer, the definition of "x" on page 22 has been chosen to suit the antenna pattern of an array of overall length "D". In this case, the magnitude of the previously defined phase change between the array and the grating elements must necessarily involve units of the basic distance $D/2$, and no change is required in the arguments of the various interference terms since the inserted factor 2 in the numerator cancels the same factor in the denominator.

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ELEMENTS OF ASTROMECHANICS

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Sproul Observatory, Swarthmore, Pennsylvania

II. The Laws of Kepler and Newton's Law of Gravitation

The derivation of Newton's law of gravitation from Kepler's law of planetary motion is a salient point in the history of astronomy, mathematics and physics. In turn, Newton's law of gravitation leads to a more general statement of the two-body problem than is contained in Kepler's three laws (Chapter III).

1. *Keplerian motion*

For any one planet the orbital motion is described by Kepler's first two laws, published in the year 1609.

The *law of ellipses*: Each planet describes an elliptical orbit with the focus at the sun, i.e.

$$r = \frac{p}{1 + e \cos v}. \tag{1.19}$$

The *law of equal areas*: The line joining planet and sun sweeps over equal areas in equal intervals of time, i.e.

$$A = \text{constant} \tag{2.1}$$

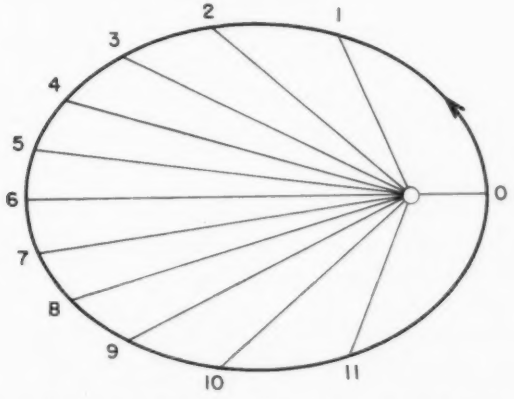

Fig. 2.1—Keplerian motion: Elliptic motion and law of equal areas with respect to focus. Orbital positions are marked for each one-twelfth of a revolution beginning with perihelion.

By combining these two laws with kinematic relations and geometric knowledge of ellipses, we are able to arrive at the clue to Keplerian motion.

2. *Velocity and acceleration components along and perpendicular to the radius vector*

Combining formulae (1.3), (1.4), (1.5)—for the areal velocity and the linear velocity components—with formula (1.19) for the ellipse in polar co-ordinates referred to the focus, Keplerian motion leads to

$$V_r = \frac{dr}{dt} = \frac{e \sin v}{p} r^2 \frac{dv}{dt} = \frac{2Ae}{p} \sin v, \tag{2.2}$$

while

$$V_\theta = \frac{2A}{r}. \tag{2.3}$$

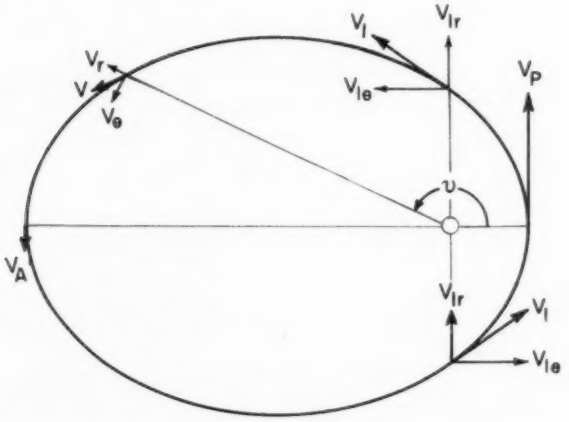

FIG. 2.2—Velocity components along and perpendicular to radius vector, specifically at perihelion, aphelion and ends of latus rectum.

The radial velocity component V_r in Keplerian motion is a simple harmonic function of the anomaly v, reaching extreme positive and negative values V_{lr} at the ends of the latus rectum and zero at perihelion and aphelion. The tangential (cross) velocity is inversely proportional to the radius vector r, reaching maximum and minimum values V_P and V_A at perihelion and aphelion respectively, and never changing sign.

Since A = constant, $dA/dt = 0$. Referring to the formula for acceleration perpendicular to the radius vector

$$f_\theta = \frac{2}{r}\frac{dA}{dt}, \tag{1.14}$$

we see that

$$f_\theta = 0. \tag{2.4}$$

Regardless of the shape of the orbit, the law of equal areas therefore implies the absence of an acceleration perpendicular to the radius vector. Hence only an acceleration along the radius vector exists, which is called *central acceleration* (*focal acceleration* might be a better name).

Since the orbits have a specific shape, namely ellipses with the origin in one focus, the specific character of the acceleration may now be found. The general formula for acceleration along the radius vector is

$$f_r = \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2. \tag{1.12}$$

Differentiating Kepler's first law leads to

$$\frac{d^2r}{dt^2} = \frac{4A^2}{pr^2}e\cos v, \tag{2.5}$$

whilst

$$r\left(\frac{d\theta}{dt}\right)^2 = \frac{4A^2}{r^3}. \tag{2.6}$$

Substitution of (2.5) and (2.6) in (1.12) yields

$$f_r = \frac{4A^2}{r^2}\left(\frac{e\cos v}{p} - \frac{1}{r}\right) = -\frac{4A^2}{pr^2}, \tag{2.7}$$

or

$$f_r = -\frac{\mu}{r^2}, \tag{2.8}$$

where

$$\mu = \frac{4A^2}{p}. \tag{2.9}$$

3. *Inverse square law*

Formula (2.8) states the famous inverse square law: for any one planetary orbit Keplerian motion implies an acceleration pointing to the

focus (sun) and varying with the inverse square of the distance r from the focus. Moreover the acceleration is proportional to a constant μ, sometimes referred to as the *inertial constant*, and which may be described as the characteristic gravitational strength of the sun-planet binary.

The constant areal velocity A, in terms of the space-time dimensions of an ellipse, is

$$A = \frac{\pi a^2 \sqrt{1 - e^2}}{P} \quad \text{whence} \quad A^2 = \pi^2 p \frac{a^3}{P^2}. \tag{2.10}$$

Here P is the period of revolution of the orbital motion.

Substitution in (2.9) yields the useful space-time expression for the inertial constant

$$\mu = 4\pi^2 \frac{a^3}{P^2}, \tag{2.11}$$

and (2.8) may be written as

$$f_r = -4\pi^2 \frac{a^3}{P^2} \frac{1}{r^2}. \tag{2.12}$$

4. *Kepler's third law. Generalization of the inverse square law*

Kepler's third, or harmonic law announced in the year 1619 states the proportionality of the cube of the mean distance, a, to the square of the period of revolution, P, for all planets, i.e.

$$\frac{a^3}{P^2} = C. \tag{2.13}$$

In comparing Kepler's third law (2.13) with formula (2.11), we note that

$$\mu = 4\pi^2 C, \tag{2.14}$$

i.e., μ is *one and the same constant* for *all* planetary orbits. The central acceleration (2.8) for all planets, at all times, may thus be written as

$$f_r = -4\pi^2 C \frac{1}{r^2}. \tag{2.15}$$

The same result is easily obtained for the simplified assumption of circular orbits. For uniform circular velocity

$$V_\theta = \frac{2\pi r}{P}. \tag{2.16}$$

Introducing Kepler's third law, $r^3/P^2 = C$, we have

$$V_\theta^2 = 4\pi^2 \frac{C}{r}, \tag{2.17}$$

for all orbits.

Also for any one orbit, the central or centripetal acceleration is

$$f_r = -\frac{V_\theta^2}{r}, \tag{1.15}$$

and substitution of (2.17) yields

$$f_r = -4\pi^2 C \frac{1}{r^2}. \tag{2.15}$$

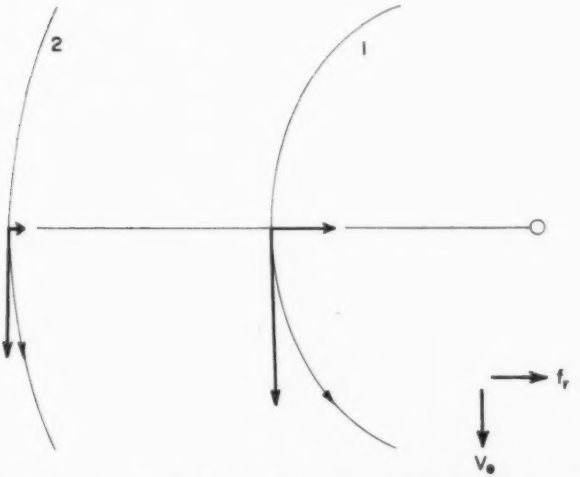

FIG. 2.3—Kepler's third law and the inverse square law for two circular orbits, whose radii are in the ratio 2 to 1. The ratio of the periods is $2\sqrt{2}$, that of the velocities $1/\sqrt{2}$. The corresponding ratio of the central accelerations is 1/4.

Formula (2.17) expresses the simple, interesting and useful fact that for the idealized case of circular motion the orbital velocities of the planets vary with the inverse of the square root of their distance from the sun.

5. *Further generalization of the inverse square law: Newton's law of gravitation*

The inverse square acceleration is now assumed to be mutual, i.e., not only does the planet "fall" toward the sun, but also the sun toward the planet, or rather, both fall from opposite sides toward their centre

of mass. Using absolute values, the observed relative acceleration f_r is considered the sum of the acceleration f_m of the planet and of the much smaller acceleration f_M of the sun, i.e.,

$$f_r = f_m + f_M \tag{2.18}$$

while the ratio of the acceleration is inversely proportional to the masses, i.e.

$$\frac{f_m}{f_M} = \frac{M}{m}. \tag{2.19}$$

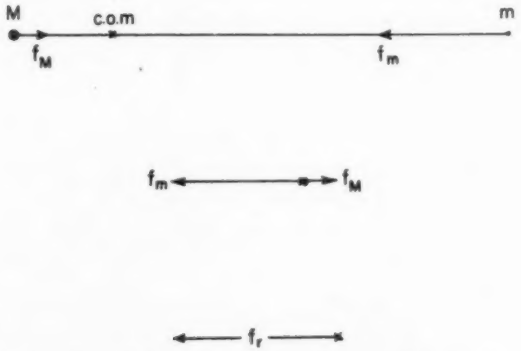

FIG. 2.4—Accelerations of planet (f_m) and sun (f_M) toward centre of mass. Mass-ratio and relative acceleration f_r of planet toward sun.

For each planet (m) and the sun (M) we have therefore

$$f_r = f_m\left(1 + \frac{m}{M}\right). \tag{2.20}$$

The acceleration f_r of planet relative to sun is therefore $(1 + m/M)$ times the acceleration f_m between the planet and the centre of mass of sun and planet. If the planets had negligible mass, f_M would be negligible and f_r would equal f_m. The mass-ratios M/m for the sun-planet binaries are very high and the centre of mass is close to the centre of the sun. The lowest ratio of masses is 1047 for the sun-Jupiter binary; the mass-ratio of the sun-earth binary amounts to 333,000.

The constant C in Kepler's third law, and hence the inertial constant μ (2.14), must vary therefore from planet to planet and be slightly in

excess of the value of this quantity which would correspond to a planet of negligible mass. The inverse square relation

$$f_r = -\frac{\mu}{r^2}, \tag{2.8}$$

is now further generalized by realizing that the mutual acceleration f_r between the two masses M (sun) and m (planet) is not only inversely proportional to r^2 but also proportional to $1 + m/M$, to allow for the minute acceleration of the sun due to the planet. We therefore write

$$\mu = \mu'\left(1 + \frac{m}{M}\right) \tag{2.21}$$

where μ' is a constant which would equal μ for a planet of negligible mass. We prefer to write (2.21) as follows

$$\mu = G(M + m), \tag{2.22}$$

and (2.8) becomes

$$f_r = -G\frac{M + m}{r^2}. \tag{2.23}$$

This is Newton's law of gravitation, which is assumed to be universally valid. G is called the *constant of gravitation* and from laboratory experiments is found to be 6.67×10^{-8} in the c.g.s. system.

The separate (absolute) accelerations of sun and planet toward their centre of mass are given by

$$f_m = G\frac{M}{r^2}, \qquad f_M = G\frac{m}{r^2}, \tag{2.24}$$

which satisfies the relations

$$f_r = f_m + f_M, \tag{2.18}$$

and

$$\frac{f_m}{f_M} = \frac{M}{m}, \tag{2.19}$$

and also the definition of gravitational force:

$$F = Mf_M = mf_m = G\frac{Mm}{r^2}. \tag{2.25}$$

Although Newton's law of gravitation is formulated for the ideal case of "particles" of mass, it can be shown that objects possessing spherical

symmetry attract each other as if all their mass is concentrated in their centres. (A proof will be given in Chapter V.) Sun, stars, planets and satellites follow this property of spherical symmetry to a high degree of approximation.

6. *Gravity and Gravitation*

The free-falling motion of a (thrown) object near the surface of the earth appears to be of the same nature as the free-falling motion of a planet around the sun or of the moon around the earth. In other words, *gravity* appears to be of the same nature as *gravitation*.

At any one location on the surface of the earth all objects fall equally fast in vacuo. This was first observed by Galileo and very accurately proven by Eötvös. At first sight, this rate of falling is the same all over the earth. This is explained by the virtual constancy of r for the nearly spherical earth and the completely negligible effect of the object's mass (m) compared with that of the earth (M). The absolute value of f_r—or as it is usually called in this case, the surface gravity g—is therefore nearly constant and may be written as

$$g = G\frac{M}{r^2}, \tag{2.26}$$

where r is the radius of the assumed spherical earth.

However, g is smaller at the earth's equator than at the poles, for the following reasons. (1) At the equator we are farther away from the earth's centre and the falling effect is therefore reduced as compared with the poles. (2) The rotation of the earth results in a slight "centrifugal effect", or "outward falling" (Chapter I) which is largest at the equator (3.37 cm./sec.^{2}), and is zero at the poles. The combined effect is a slight variation in the rate of falling, or gravity, ranging from 983 cm./sec.^{2} at the poles to 978 cm./sec.^{2} at the equator or, more precisely, expressed by the formula of Helmert (1896)

$$g = 983.2 - 5.2\cos^2\phi. \tag{2.27}$$

The *weight* W of an object is the local force of gravity, i.e.,

$$W = gm, \tag{2.28}$$

or substituting (2.26), at the surface of an assumed spherical earth:

$$W = G\frac{M}{r^2}m. \tag{2.29}$$

To clinch the identity of gravity and gravitation, we turn our attention to the moon, the nearest cosmic object, which continuously falls around the earth, and which we may also regard as the most distant object connected with the earth. Assuming a circular orbit with radius 384,400 km., and sidereal period of 27.32 days, we find an orbital velocity of 1.023 km./sec. for the moon. A simple calculation, using formula (1.15), shows that the moon falls to the earth at the gentle rate of 0.272 cm./sec.^{2}. This is only 1/3600 of the falling acceleration g of objects near the surface of the earth. Now the moon is 60 times as far from the (centre of) the earth as objects near the surface of the earth, and $60 \times 60 = 3600$. This agrees beautifully with the inverse square law, and is a convincing argument for the identity of gravity and of gravitation.

The same identity is, of course, clearly exhibited by the orbital motions of the artificial satellites first launched around the earth on 1957 October 4.

7. *Masses of celestial objects*

The mass M of any celestial object may be measured by the absolute value f of its gravitational effect $f = GM(+m)/r^2$ on another object of (negligible) mass m. The mass is thus obtained by the simple relation

$$M(+m) = \frac{fr^2}{G}. \tag{2.30}$$

The earth's mass is thus determined from accurate measurements of the gravitational acceleration g on its surface. Assuming spherical symmetry for the earth, a radius $r = 6.371 \times 10^8$ cm., and a surface gravity of 980 cm./sec.^{2}, we find

$$M_{\text{earth}} = 5.98 \times 10^{27} \text{ gm.}$$

The alternative formula relating the masses of an astronomical binary system with its space-time dimensions is found by combining the formula for the inertial constant

$$\mu = 4\pi^2 \frac{a^3}{P^2}, \tag{2.11}$$

with its value in Newton's law of gravitation

$$\mu = G(M + m) \tag{2.22}$$

from which follows

$$M + m = \frac{4\pi^2}{G} \frac{a^3}{P^2}. \tag{2.31}$$

The sun's mass is thus found from knowledge of the mean distance (astronomical unit) $a = 1.496 \times 10^{13}$ cm. and of the sidereal year $P = 365.2564$ mean solar days $= 3.1558 \times 10^{7}$ sec. We thus find, neglecting the earth's mass,

$$M_{\text{sun}} = 1.99 \times 10^{33} \text{ grams}$$

or 333,000 times the earth's mass.

Formula (2.31) is particularly useful when the mass m is not negligible compared with M, as for example in the case of the earth-moon system or for stellar binaries (Chapter VI).

In many problems it is convenient to introduce the astronomical units of space, time and mass, i.e., the astronomical unit of distance, the sidereal year and the mass of the sun. Comparing any astronomical binary with the sun-earth binary it is easily seen that

$$M + m = \frac{a^3}{P^2}, \tag{2.32}$$

if the astronomical units of space, time and mass are used. This is the well-known *harmonic relation.*

The determination of the masses of the separate binary components involves the determination of their centre of mass. In the earth-moon binary for example, the moon's mass is thus found to be 1/81.3 times the earth's mass. In stellar binaries the components generally have the same order of mass (Chapter VI).

(*To be continued*)

CANADIAN SCIENTISTS REPORT-XXI

ASTRONOMY IN CANADA

In order that readers may get a comprehensive view of research programmes at present being undertaken by Canadian institutions, the Journal is publishing in this and the next issue abstracts of the reports recently read to the National Committee for Canada of the International Astronomical Union—Editors.*

Dominion Observatory

Positional Astronomy Division—Report by Malcolm M. Thomson

Astronomical observations are made with: meridian circle, photographic zenith telescope, Markowitz moon camera, satellite camera. The electronic equipment includes: quartz crystal oscillators, facilities for time and frequency comparisons, facilities for disseminating time and frequency, auxiliary test and maintenance equipment.

The activities can be summarized under six headings.

1. Meridian Circle programmes. There are at present two programmes in progress both of which are international in scope, being shared by meridian circle observers at some fifteen or more locations around the world. The first is a programme of 1503 FK3 supplementary stars requiring 6 observations per star. The second is a programme of 3754 AGK3 reference stars required for the present photographic survey which will provide improved positions and proper motions for stars down to the 9th magnitude. Along with both these programmes, the stars of the FK3 are also observed and this will contribute to the planned revision of the fundamental catalogue.

2. Observations with the PZT which are made each clear night averaging about 190 nights per year, provide direct evidence of astronomical time, the variation of latitude, and the relative corrections to the catalogue of the Ottawa PZT stars. Weekly summaries of observed latitude and monthly summaries of UT2 determinations are reported to the BIH, the ILS, and others.

A caesium resonator, designed, built and operated at the National Research Council, is referred daily to the Observatory time service,

*In Ottawa, Nov. 27, 1959.

providing direct evidence of variations in astronomical time resulting from changes in earth rotation.

3. The national time service depends upon the careful maintenance of several precision quartz timekeepers, the assessment of clock rates with respect to UT2, and the communication of time signals to Canada and abroad by various means. Monthly summaries of corrections to broadcast time signals, together with an assessment of signals received from abroad, are distributed to all interested institutions and persons. Signals from the U.S. Naval Observatory and from WWV have been used to compare the Ottawa PZT catalogue with outside sources, and also to note the travel time of radio waves.

4. A co-operative effort is under way to determine the relation between UT2 and Ephemeris Time by direct lunar photography using the Markowitz Moon Camera. One such is being operated in conjuction with the 15-inch telescope, and the plates, after identification of the star field, are sent to Washington for measurement and reduction. Some plates have been measured at Ottawa, but due to staff limitations have not had a thorough reduction.

5. Predictions for satellite observing have been carefully followed. Favourable passages of Sputniks have been photographed using equatorially-mounted cameras equipped with rotating shutter. The scaled results have been forwarded to Cambridge, Mass., Slough, England, and Moscow, U.S.S.R.

6. A major instrumental development in the form of a mirror transit telescope has been pursued during the past six years. The instrument is in an advanced stage with various components undergoing tests. It is expected that preliminary observations will commence in the first half of 1960.

Stellar Physics Division—Report by Ian Halliday

The astronomical research programmes are in four main fields: radio astronomy, solar physics, meteor astronomy, meteorite craters.

Radio Astronomy. An 84-foot radio telescope has been erected in an isolated valley about 12 miles south of Penticton, B.C. Construction of the associated buildings is essentially completed and much of the electronic equipment has been received. Some months will be required for testing of components and for mounting auxiliary equipment on the telescope. A small transmitter located on a mountain top about seven miles away will be used to assist in determining the antenna pattern of the dish.

The first observing programmes with the telescope will be measurements of hydrogen radiation at 21 cm. These include a search for the

Coma cluster of galaxies, mapping of the area near M31, a search for certain elliptical galaxies and probably an independent search for 21-cm. absorption in Cygnus A. Later, it is hoped to have auxiliary equipment to search for the Zeeman effect at 21 cm. in the Cassiopeia A radio source.

It is expected that a suitable parametric amplifier for work at 21 cm. can be developed and tested on the telescope during 1960.

Solar Physics. The Hα flare patrol has continued on a reduced scale since the end of I.G.Y. Some 200 flares observed during I.G.Y. have been reduced photometrically to obtain the curves of growth and decay.

A study of the wave-length displacements between the limb and centre of the solar disk has been completed. Absorption lines of three different atoms were studied in the photographic infrared region of the spectrum.

An analysis of the rotation-vibration bands of the CN molecule in the near-infrared is also nearing completion.

A solar magnetograph is being constructed and may be in operation late in 1960. The emphasis will be on rapid, automatic plotting of the magnetic contours in isolated areas of the solar disk. Preliminary plans have been made for a new solar telescope with an aperture of about 24 inches.

Meteor Astronomy. The Meanook and Newbrook Meteor Observatories are in routine operation. The two Super-Schmidt telescopes provide pairs of meteor photographs from which changes in upper atmospheric densities may be studied. Routine reduction of these films has been started recently at Ottawa. Seventeen grating spectrographs are in operation at the two stations. Analysis of the better spectrograms is in progress at Ottawa.

A co-operative programme of meteor observing between the Observatory and the National Research Council was carried out during I.G.Y. and 1959, at Springhill, near Ottawa. Some good spectra were secured and are being studied. In addition a large programme of visual meteor observations was conducted at Springhill and reductions are well advanced.

Meteorite Craters. An intensive search of aerial photographs of Canada has revealed a number of circular features, some of which may be the remains of meteorite craters. Several of these features have been studied by geophysical techniques and in two cases drilling operations have been conducted to bring up rock cores from considerable depths. It is considered highly probable that the craters at Holleford, Ontario, and Deep Bay, Saskatchewan, are due to meteoritic impact. Investigations are continuing on most of the suspected crater formations.

In conjunction with the crater search a theoretical study has been undertaken of the phenomena associated with an intense meteoritic

explosion. Previously such work was based on inference from old craters and an extrapolation of experimental values to high pressures. As a result of the release of data on confined nuclear detonations it is expected that the problem can be resolved by experimental values of shock propagation, peak pressure and ground motion.

Queen's University

Report by G. A. Harrower

A decision made three years ago—to establish at Queen's a programme of astronomical research—has resulted in progress since that time which may be summarized as follows: New laboratory space of about 3000 sq. ft. has been built on the campus; this includes an observatory, library and seminar room, dark room, two work rooms, and office space for six graduate students and two members of the staff. The observatory houses a new telescope, just installed. It consists of a 15-inch yoke-mounted reflector with 3-inch finder and 6-inch catadioptic auxiliary telescopes, camera attachment, grating spectrograph, 3-inch astrograph, and a photoelectric photometer. Our need will be for a full-time worker in this field, preferably a post-doctoral fellow, experienced in the use of a small or moderate telescope with spectrographic and photoelectric attachments.

Radio astronomy is carried out at the field station located several miles from the city. A small building and approximately 12 acres of land provide space for a variety of apparatus. A programme of measurements of the scintillation of four radio sources (Cassiopeia A, Virgo A, Taurus A, and Cygnus A) has produced results relating the phenomenon to the terrestrial ionosphere, rapid variations of the earth's magnetic field, and Type II radio bursts from the sun. Work now in progress concerns long-period regularities in the scintillation records, and the extent and rate of change of ionosphere diffraction patterns. A second programme, aimed at source counting with cosmological implications, has now reached the stage of construction of an antenna and receiver to operate at 830 Mc./sec. The antenna is a switched cross-type of interferometer designed to have a beamwidth of 0.75° and intended to cover the northern half of the celestial sphere. The intention is to measure location, intensity, and diameter of cosmic radio sources, the last using a compound interferometer with a 3000 wave-length base-line. The problem of designing this system has led to three theoretical studies which deal respectively, with the theory of large antenna design, the optimum relation between resolution and sensitivity in a radio telescope, and the interpretation of Australian measurements in terms of density, inherent intensity and distance of extragalactic radio sources.

The work of the astronomy group has been shared by members of the staff of four departments including Dr. A. V. Douglas (Mathematics), Dr. R. M. Chisholm (Electrical Engineering), Mr. E. L. Dauphin (Civil Engineering), and Dr. G. A. Harrower (Physics). To date, eleven graduate students have participated in the programme.

David Dunlap Observatory—University of Toronto

Report by John F. Heard

In astronomy at the University of Toronto we are encountering a marked enrolment increase, 50 per cent. this year over last year in the eleven undergraduate courses which we offer. Whereas a few years ago two or three students in the Mathematics and Physics course would elect astronomy in fourth year, this year there are twelve taking the fourth year astrophysics course, and of these five are taking the full graduating option in astronomy. At the moment we have six graduate students enrolled in astronomy. To these we are giving four graduate courses and directing their research.

Two Ph.D. theses have been accepted during the past month; Pierre Demarque, directed by Searle, has studied the interior structure of subdwarfs. He has constructed models of homogeneous stars with masses of 1.0, 0.8, and 0.6 suns and a variety of compositions. The results are that the positions of the main sequences for different compositions (varying from $Y = 0.0$ to $Y = 0.175$ and $Z = 0.005$ to $Z = 0.1$) differ by less than 0.3 mag. from one another in the *theoretical* HR diagram and the effects of metal deficiency and helium deficiency tend to cancel one another out. "Subdwarfness" is therefore something which is only observed in the colour-magnitude diagram and is an effect of differential blanketing.

Gustav Bakos, directed by MacRae, has made careful photometric and spectrophotometric studies of 75 visual binaries, fixing the absolute visual magnitudes and the colours of both components. From the C-M diagram he has been able to assess the ages of many of the systems in the light of current views on stellar evolution. Some are so young as to be still in the stage of gravitational contraction while others appear to be as old as 10^{10} years.

The claims of an expanded teaching department and the varied research interests of members of our staff and our students have had an effect on the radial velocity work to which some years ago each of us gave a portion of his time. No new large programmes have been laid out in the past few years.

Following is a summary of velocity programmes uncompleted or unpublished.

(a) The radial velocities of 55 supposed members of the Alpha Persei cluster—completed, unpublished.

(b) Velocity curves from 163 spectrograms for five cycles of the β Cephei-type variable 12 Lacertae in September 1956 have been forwarded to de Jager to be correlated with other velocity and photometric data.

(c) Observations and measurements have been nearly completed for 55 Hins Catalogue stars in Kapteyn areas in the north galactic polar cap.

(d) Radial velocities of two hundred stars of type A0 and later in extensions of the Kapteyn areas awaits publication.

(e) A velocity programme involving 110 faint very distant B-type stars, some of them members of aggregates, is progressing slowly.

(f) Miss Northcott and Heard continue to study eclipsing and spectrographic binaries. Six, published in abstract only, and two others, await publication in full, and six others are being worked on. Several of these have particular interest.

Instrumental developments in hand include: design of a new grating spectrograph for the 74-inch telescope for which a National Research Council grant has been received; improvements in the photoelectric spectrophotometer for the 74-inch and the three-colour photometer for the 19-inch.

Report by Helen Sawyer Hogg

The programme of which I wish to report, on globular clusters in general and variable stars in them in particular, has been a continuing programme for almost thirty years. I began it at the Dominion Astrophysical Observatory in 1931, as a carry-over from my Ph.D. field at Harvard Observatory. Since that date I have acquired about 4500 direct photographs of globular clusters with four different telescopes, the Dominion Astrophysical 72-inch (300) the David Dunlap 74-inch (2400) and 19-inch (1500), and the Steward Observatory 36-inch (300), to supplement the northern plates. All these plates are presently housed at the David Dunlap Observatory. From this extensive plate material over 40 research papers on various aspects of variables in clusters, or on clusters themselves, have already been published and many plates still remain to be studied.

The programme primarily was to hunt for variables in globular clusters which had never been searched, and once the variables were found, to determine their periods. Several hundred variables have been so found, and a similar number of periods determined, but not always on a one-to-one correspondence however, because for two of the clusters I found

richest in variables, NGC 6402 and NGC 6934, I have not yet published periods. The variables investigated range in period from the shortest period RR Lyrae stars, period around 1/3 of a day, to an RV Tauri star with period of 90 days. They provide information on distances of globular clusters and on population II stars.

An outgrowth of the observational programme was an allied bibliographical programme, the first fruits of which were the publication in 1939 of a catalogue of 1116 variable stars in globular clusters. A second edition of this catalogue, with 1421 variables, was published in 1955. The publication of the first catalogue showed a need for co-ordinating all published observations on individual globular clusters, and not merely those concerned with variables. Accordingly "A Bibliography of Individual Globular Clusters" was published in 1947 giving all references, from earliest times, which mention an individual cluster. The first supplement to this bibliography has occupied much of my time this year, and should appear early in 1960. Material from this bibliography on both globular and galactic clusters has also recently been published in collaboration with Dr. George Alter of Prague.

Report by Leonard Searle

Since the completion of work on the recombination spectrum of nebular hydrogen my efforts have been primarily devoted to the high-dispersion analysis of hot carbon stars. In the study of R Coronae Borealis, a differential curve of growth analysis has made possible the determination of the carbon/hydrogen ratio, the carbon/iron ratio and the rare earth/iron ratio. A comprehensive study of opacity sources in hydrogen-poor stars has shown that the photo-ionization of neutral carbon is probably responsible for the opacity in the atmosphere of R CrB.

A differential abundance study is being carried through for the two carbon stars H.D. 182040 (H poor, C^{13} poor), H.D. 156074 (C^{13} rich) and a comparison standard β Her. It is expected that an estimate of opacity and a fairly reliable rare earth/iron ratio will be obtained.

Spectra have been obtained at 33 A./mm. with the 74-inch telescope of 100 high-velocity G dwarfs and subgiants. An attempt is being made to apply a modification of Oke's luminosity classification to these stars and to effect an objective three-dimensional spectral classification.

The scanner is being modified to allow refrigeration of the photomultiplier tube and an electrometer preamplifier is being constructed. This will allow an extension of scanning work to 11th magnitude with a 10-A. band pass. It is hoped to add a red-sensitive photomultiplier to the scanner soon. Among the problems so far studied which we are

continuing are: (1) Population classification; (2) Continuous energy distribution of H-poor stars; (3) Continuous energy distribution in Cepheids and RR Lyrae stars.

Report by S. van den Bergh

During an investigation of galaxies, using the prints of the Palomar Sky Survey, it was found that the appearance of spiral and irregular galaxies depends on their intrinsic luminosity. The luminosity of a galaxy may be estimated on these prints with an accuracy of 0.5 magnitudes. The results of this investigation may be summarized as follows.

1. A large nearby cluster of galaxies centred in Canes Venatici has been isolated using luminosity classification techniques.

2. The mass obtained for this cluster, which consists of late-type galaxies, using the observed velocity dispersion and the virial theorem, exceeds the combined mass of the constituent galaxies by at least one order of magnitude.

3. Using luminosity classification techniques the absolute magnitudes of supernovae have been recalculated. The mean magnitudes at maximum are: Type I, $M_{pg} = -18.7 \pm 0.3$ (m.e.); Type II, $M_{pg} = -16.3 \pm 0.3$ (m.e.).

4. Assuming a Hubble constant H = 100 km./sec./mpc. to calibrate the luminosity classes the distance modulus of the Virgo cluster is found to be $30.11 < m - M_0 < 30.46$ indicating that the relative radial velocity of the local group and the Virgo cluster, which is superimposed on the systematic redshift, is small.

5. The luminosity classification of members of the local group makes it possible to obtain an absolute calibration of the extragalactic distance scale. Using the distances of local group members recently given by Sandage one obtains H = 100 (+20, −17) (m.e.) km./sec./mpc.

Report by Donald A. MacRae

Research in radio astronomy is being carried out jointly by the Departments of Astronomy and Electrical Engineering. In the latter department the chief responsibility now lies with Professor J. L. Yen, who has made this his principal research activity.

The work of the radio astronomy project is built around a programme of graduate studies. At the present time we have four graduate students who are working full time in radio astronomy, and two others whose work is closely associated with it.

Our absolute flux work is carried out by means of the total power radiometer, the horn antenna, and various other antennas in combination with the horn. The significant feature of the horn is our ability to calculate

its gain from its geometry. A measure of the flux from the sun generally is made daily at noon. The horn is also used for the temperature calibration of selected sky points in accordance with recommendations of Commission 40 of the I.A.U. To measure a discrete source signal against the general galactic background two other parabolic cylinder antennas are employed. Each is used with the horn and with each other to provide interferometer records.

One of the principal problems we encounter in absolute flux measurements is the calibration of the radiometer by means of laboratory sources of known power. For this we rely on Nyquist's theorem and thermal sources. Mr. Duncan McNeill, a graduate student, is engaged in this development. He has built one source to be used at liquid nitrogen temperature and one at about 800°K.

Antenna design is the current responsibility of Mr. Robert Vicary. Working from a theoretical study by a former graduate student, D. Sengupta, he spent the summer designing and testing a long Yagi, determining its pattern and the effects of stacking such elements in an array. The large effective area of a single element permits considerable separation between elements. The long Yagi is simple to construct, and offers simple means of phasing. A test array at 700 Mc./sec. containing 64 elements is currently being constructed at the Observatory. The same design may be used for a larger array at the Algonquin Park site. The prototype itself can be used in various ways for survey work and studies of galactic HII regions at its design frequency.

Work on a parametric preamplifier using diffused-junction diodes went forward vigorously during the summer and fall in the hands of Mr. Sergei Dmitrevsky and Mr. Bruce Cherrington. Two working models have been constructed and performance has been in accordance with theoretical expectation. A third design, representing successive improvement, is now being fabricated. Future plans include the development of a parametric amplifier at 1420 Mc./sec. and an investigation of its behaviour and capabilities, especially as regards stability. This instrument may find application at the Dominion Radio Astrophysical Observatory.

Mr. David Hogg's interest lies in galactic HII regions at short wavelengths. He is currently planning an observational programme to be carried out early in the new year on certain selected regions of the Milky Way.

As this brief review demonstrates, the research work in radio astronomy at Toronto is varied in nature in order to give scope to the interests and abilities of the graduate students and at the same time to enable them to attack problems of significance. Fortunately, plans are on foot to provide additional instrumental facilities at the Algonquin Radio Observatory.

NOTES FROM OBSERVATORIES

Dominion Astrophysical Observatory, Victoria, B.C.

Dr. E. H. Richardson joined the staff on September 1, 1959 and Mr. P. E. Argyle left the Observatory on August 14, 1959 to join the staff of the Dominion Radio Astrophysical Observatory at Penticton. Dr. A. H. Batten of the University of Manchester came to the Observatory on September 28, 1959 as a National Research Council postdoctoral fellow. He intends to make spectroscopic observations of a number of early-type eclipsing binaries in order to improve our knowledge of the dimensions of these systems. Three members of the staff are on education leave for the 1959–60 university term: Mr. E. K. Lee at the University of Michigan, Ann Arbor; Mr. B. Caner of the geomagnetism section at the University of Alberta; and Mr. W. R. H. White of the seismology section at the University of British Columbia.

In October the Minister of the Department of Mines and Technical Surveys, the Hon. Paul Comtois, and the Deputy Minister, Dr. Marc Boyer, visited the Observatory. Other visitors from Ottawa included Col. R. E. Denison, Chief of Property and Building Management Division, and Mr. D. E. Macdonald, Assistant Chief of Personnel. In November the Astronomer Royal, Dr. Woolley, and Dr. Hunter from the Royal Greenwich Observatory were visitors, and in December Dr. L. Biermann, Director of the Astrophysical Section of the Max Planck Institute, Munich, Germany.

In January 1960, Professor John Climenhaga of Victoria College completed the requirements for the awarding of a Ph.D. in Astronomy at the University of Michigan. He obtained spectrographic observations for his thesis topic at this Observatory during the summers of 1956, 1957 and 1958. It will interest the readers of the Journal that some of the magnitude observations of RU Cam used in his thesis were made by Mr. Norman Hutchings of the Victoria Centre.

The Observatory was represented at two meetings of the American Astronomical Society—in September at Toronto by K. O. Wright and E. H. Richardson, and in December at Cleveland by E. H. Richardson. In November R. M. Petrie attended the meeting of the Board of Directors of the Astronomical Society of the Pacific in San Francisco and the meeting of the National Committee of the I.A.U. in Ottawa.

REVIEW OF PUBLICATIONS

Elementary Astronomy by Otto Struve, Beverly Lynds and Helen Pillans. Pages 396; 7 × 10 in. New York, Oxford University Press, 1959. Price $7.00.

The authors state that this book is not intended to replace the many excellent text-books of descriptive astronomy now in use, but rather to fill the needs of those students who require a general background of astronomy for the study of physical science. In order to relate astronomy to the main ideas of physical science, the authors admit to having sacrificed some descriptiveness. They have not, however, sacrificed illustrative material; the book is profusely illustrated with many fine diagrams and superb astronomical photographs, many of them full-page in size, and four large star charts.

Those who have followed Dr. Struve's long series of articles in *Sky and Telescope* will recognize his style throughout many of the 33 chapters which range in content, in the traditional order, from the earth and its motions through the solar system, sun, stars, Milky Way and galaxies, with telescopes and accessories tacked on, rather surprisingly, at the end. The most notable characteristic of Dr. Struve's style is the continued emphasis on simple arithmetical calculations to relate quantitatively the astronomical ideas with one another and with physical principles. The authors make these computations at every possible juncture, often with a good deal of ingenuity and always effectively. Yet they have avoided formulas to a large extent, and they have avoided completely not only calculus but also logarithms and trigonometric functions—sometimes at the cost of some awkwardness. Many teachers, while rejoicing in the clear explanations and in the emphasis on the quantitative, will regret that basic formulas like $M = m + 5 + 5 \log p$ could not have been included at least in footnotes or appendices.

Elementary Astronomy contains an amazing amount of information, and although it has the character of a text-book rather than of a popular book, it is surprisingly easy to read. As an up-to-date reference book it could be recommended highly to a general reader who is not mathematically trained, providing his resistence to mathematics does not include arithmetic and the simplest algebra and geometry.

J. F. HEARD

Atlas Eclipticalis 1950.0 by Antonín Bečvář. Charts 32 plus pages iv, 26 × 19 in., cloth bound. Nakladatelství Československé Akademie věd Praha, 1958. (Can be ordered through Sky Publishing Corp. Harvard College Observatory, Cambridge 38, Mass.) Price $17.00.

This is the latest atlas from Czechoslovakia, a country which has gained an international reputation for the excellence of its astronomical charts and star maps. A high quality of design and printing has been maintained in this current publication.

The atlas covers the equatorial regions of the sky from declination 30°N. to 30°S. and thus includes all of the ecliptic region as well. Each individual chart extends over 30° of declination and 1.5^h or 22.5° of right ascension, the scale being 1° = 2 cm. The epoch of 1950.0 is used for the co-ordinate system. Parallels of declination are drawn every 5 degrees and meridians of right ascension every 30 minutes. A transparent grid gives co-ordinate lines every half degree of declination and every minute of right ascension. The atlas is complete down to approximately visual magnitude 9.0 and contains stars in special categories down to visual magnitude 10.0. Stars are represented in half-magnitude classes.

A unique feature of the atlas is the six-colour printing which makes it possible to identify the major spectral classification of any star. The colours violet, blue, green, yellow, orange and red are used for the six groups of spectral classes OB, A, F, G, K, and MNRS respectively. The colours are quite clear and distinct even for the faintest stars shown.

Double stars, variable stars, bright star designations and numbers, and constellation boundaries are all indicated by appropriate symbols. We have had the atlas in use for some time now at the National Research Council and have found it extremely convenient for identifying star fields and checking on faint objects. I can recommend it to anyone engaged in photographing the equatorial portion of the heavens. It is to be hoped that some day a similar atlas will be produced for the polar areas of the sky.

PETER M. MILLMAN

The Sky Observer's Guide, A Handbook for Amateur Astronomers by Newton Mayall, Margaret Mayall and Jerome Wyckoff. Pages 125; 6¼ × 9 in. New York, Golden Press, Inc., 1959. Price $2.95.

The Sky Observer's Guide is a very practical, readable and attractive book for people interested in observing the heavens. Not only does it discuss in some detail the interesting objects in the sky, when and where

to find them, but it also gives valuable information about binoculars, telescopes and accessories. It contains an important chapter about the care of equipment. The book is profusely illustrated with photographs as well as with paintings and drawings by John Polgreen. The star maps are for the epoch 1900, and show the stars to 5th magnitude in colours representing the spectral classes.

The name of Margaret Mayall is well known to readers of the JOURNAL as the author of the section "Variable Star Notes". It will be remembered that she and her husband recently revised the popular *Field Book of the Skies* (reviewed in this JOURNAL, vol. 49, p. 135, 1955). The third author, Jerome Wyckoff, has edited many popular books on science and is an active member of the A.A.V.S.O.

The Sky Observer's Guide is a book which can be highly recommended to all observers, both young and old.

RUTH J. NORTHCOTT

Light Scattering by Small Particles by H. C. van de Hulst. Pages 470; 9 × 5¾ in. New York, John Wiley and Sons Inc. and London, Chapman and Hall Ltd., 1957. Price $12.00.

The scattering of electromagnetic waves by obstacles of various sizes, shapes, composition and distribution is a very complicated phenomenon. Classical physicists of the last century spent a great deal of effort on the subject resulting in a reasonable understanding of the nature of scattering and the art of predicting some of its more important aspects. This quite often required a judicious combination of physical reasoning and more rigorous but intangible results from solutions of Maxwell's equations. In recent years further clarifications and refinements were stimulated by the development of radar and microwave technique. The result is an enormous amount of material available on the subject. This book contains a comprehensive account of both old and new works on scattering responsible for the extinction of light waves through a propagating medium in which each obstacle scatters independently and by itself.

The book is divided into three main parts. Part I comprises five chapters dealing with the basic scattering theory for particles that may have arbitrary size, shape and composition. A scattering matrix depending on the direction of incidence and of scattering is used to characterize any finite particle. This is then applied to a medium of independent particles. The major portion of the book is in Part II, where particles of special types are discussed in twelve chapters. It opens with general

descriptions of the two limiting sizes of particles, namely, small or large compared with wave-length, and those with refractive index close to unity, all of arbitrary shape. This is followed by the most comprehensive treatment of scattering by a sphere that has appeared anywhere. The part is then concluded by particles of some other regular shape and large bodies with a smoothly curved surface. The last part discusses applications to chemistry, meteorology and to astronomy. Of the last, we can mention planetary atmospheres, interplanetary dust and solid grains in interstellar space.

Whenever possible, the exposition is based on lucid physical reasoning. When this is unavailable, the more complicated rigorous solutions of Maxwell's equations are used. Suitable approximations are then introduced to simplify the more complicated mathematical expressions into tangible forms. These are backed by a profusion of tables of numerical values and graphs, a large portion of which have not appeared elsewhere. Perhaps due to space limitation, a few of the important formulae are quoted without derivation, while many refinements only receive cursory comments. The situation is partly saved by the excellent compendium of references included at the end of each chapter. However a novice may have to look up references more often than he intended. In this sense the book is not quite suitable for someone to learn the subject. For those who wish to apply the results of classical scattering theory, the tables and graphs and the many special cases considered should be of enormous value. Finally, experts in scattering theory will appreciate the many subtleties and comments so refreshingly presented.

J. L. YEN

NOTES

COMET BURNHAM 1959k

Comet Burnham is predicted to be about third magnitude during April, fading rapidly in May (H.A.C. 1467). At the beginning of April it is in Aquarius, rising about an hour before sunrise. It moves rapidly northward and by the 26th is visible all night in Cygnus. It passes through the bowl of the Little Dipper about April 30, and through that of the Big Dipper about May 4. The following are the predicted co-ordinates of the comet.

Date	R.A.(1950)	Dec.	Date	R.A.(1950)	Dec.
Apr. 16	$22^h\ 18.4^m$	01° 46′	Apr. 30	$15^h\ 29^m$	77° 39′
Apr. 22	21 59.1	20 54	May 4	11 43.9	59 48
Apr. 26	21 17.3	50 19	May 10	11 07.2	44 24

VARIABLE STAR NOTES

By Margaret W. Mayall
The American Association of Variable Star Observers, Cambridge, Mass.

Miscellaneous notes on variables observed by members of the A.A.V.S.O. during 1959.

001046 *X And.* A 10th magnitude maximum was observed in mid-October, following one about a magnitude brighter in October 1958.

001032 *S Scl.* A bright maximum was observed in December, about the same magnitude (6 1/2) as the 1957 one. The 1958–59 maximum was a magnitude fainter.

001838 *R And.* Alternating bright and faint maxima seem to occur regularly.

005840 *RX And.* This Z Camelopardalis-type variable was almost constant at about 11.5 magnitude from December 1958 until December 1959, when it began to vary between 11.5 and 13.

011025 *Z Psc.* More observations are needed of this bright, very red (spectrum N) semi-regular star. It varies from about 7.1 to 7.7.

013238 *RU And.* Range of variation decreased to about 1½ magnitude in 1959.

020657 *TZ Per.* More observations are needed of this Z Cam-type star. It is a difficult object to observe, and requires a large instrument. Range is from about 12 to 14.

021024 *R Ari.* Four bright maxima have been observed in succession, 2 in 1958 and 2 in 1959.

021403 *o Cet.* Maximum was observed about August 17, magnitude 3.4.

021558 *S Per.* This semi-regular variable reached a deep minimum (near 11 mag.) about the first of the year, rose to 9½ in August, then dropped to near 10½ by the last of the year.

032443 *GK Per* (*Nova 1901*). Slowly varies between 12.6 and 13.2.

040053 *XX Cam.* An R CrB-type variable, shows slow variations between 7.1 and 7.5.

043274 *X Cam.* The minima of this star show greater differences than the maxima. Normal minimum is about 13th magnitude. In 1958 the minima had depths of 14 and 12.7; in 1959 they were 12.5, 11.0, and 12.0.

053005 *Orion nebula region.* A chart of the region with a list of variables and a discussion of some of them is published in *Sky and Telescope*, page 152, January 1960. A new A.A.V.S.O. chart is in preparation; this will contain more than 50 variables. The first variable to be discovered in the region, T Orionis, has been inactive for some time, but near the end of 1959 it began to show signs of activity, with variations of 1½ to 2 magnitudes.

053326 *RR Tau.* After about half a year of inactivity, this RW Aur-type star began to vary over a larger range of brightness early in 1959. In the latter part of the year it varied from 10½ to 13½ mag.

054319 *SU Tau.* After more than 2 years with little variation, this R CrB-type star began to decrease the first of March 1959. At the end of the observing season, it had reached 14th mag. When it was again observed in August, it was about 12½, and was brightening. During November and December it was fairly steady at 10th mag.

063462 *RR Pic* (*Nova 1925*). Nearly constant at 11.6.

072609 *U Mon*. For several years this RV Tau-type star has varied from about 7 to 7½, but early in 1959 it rose to 6th magnitude and varied between 6 and 7½. In the fall observing season, it ranged to a little brighter than 6.

080835 *CP Pup* (*Nova 1942*). Constant at 11.1 magnitude.

081473 *Z Cam*. This star had a very unusual maximum in 1959. It was nearly constant around 12th mag. until the middle of February, when it began to brighten. In about 10 days it reached a maximum of 10.5 and started a very slow decrease. About 40 days later, early in April, it reached 12½ magnitude, and then varied the rest of the year from 11 to 13, with a periodicity of about 20 days.

095814 *RY Leo*. During 1958 this star varied between 10 and 11; early in 1959 it became one magnitude brighter at maximum and varied between 9 and 11.

104620 *V Hya*. The 1958–59 minimum was down to nearly 11th magnitude, the faintest it has been since 1945.

123160 *T UMa*. A faint maximum (8.5) the first of May followed a brighter one (7.5) in 1958.

132706 *S Vir*. A 6.5 magnitude maximum the middle of August followed two 7.5 magnitude ones in 1957 and 1958.

134536 *RX Cen*. A very faint 11½ magnitude maximum in the middle of November 1957 was followed by one two magnitudes brighter early in October 1958.

140512a *Z Vir*. An 11.2 magnitude maximum the last of April followed one brighter than 10 the middle of June 1958.

154428 *R CrB*. Constant since June 1956.

155526 *T CrB*. Varies about 0.3 magnitude.

155823 *RZ Sco*. Range less than 2 magnitudes during 1959; it was 4 magnitudes in 1957, and about 3 in 1958.

172486 *S Oct*. An 8th magnitude maximum in the latter part of January was followed by a 10th magnitude one in the middle of October.

174406 *RS Oph* (*Nova 1898, 1933, 1958*). At the beginning of the 1959 observing season (February), the nova was about 12.5. In April it was fluctuating between 11 and 12, and by September it increased its range and had rapid fluctuations between 10 and 12.

180222 *VX Sgr*. This semi-regular variable had an 8th magnitude maximum in October 1958, followed by a minimum at 11 in September 1959.

180445 *DQ Her* (*Nova 1934*). Fluctuated around 14th magnitude.

180911 *V849 Oph* (*Nova 1919*). Fainter than 13th magnitude.

184205 *R Sct*. No deep minimum was observed during the year. It varied between 5 and 6 with 3 shallow minima at 6.2 or 6.3.

185851 *BL Tel*. The following minima of this long period (778^{d}) eclipsing variable have been observed by Reginald P. de Kock: J.D. 2433930, 10.4; 4690, 8.9; and 6252, 9.5. The next minimum should be due in April 1960.

191019 *R Sgr*. A very bright (6.7) maximum early in November followed a faint, 8th magnitude one the last of February.

191033 *RY Sgr*. This R CrB-type variable fluctuated between 6th and 7th magnitude throughout the year.

195553 *V476 Cyg* (*Nova 1920*). Remains about 16.2 magnitude, according to Claude Carpenter and Thomas Cragg.

201520 *V Sge*. Frequent fluctuations of half a magnitude or more are observed. In June 1959 the fluctuations were between 12.5 and 13. The star gradually brightened

until it was between 11.3 and 12.3, where it remained during July and August. After a drop of half a magnitude, it again increased to about 10th mag., and fluctuated between 10 and 11.5 through November. In December it dropped from 11.5 to 12.5. A report will be made later of the results of the special observing programme set up by the A.A.V.S.O. during the summer and fall of 1959.

204318 *V Del.* A bright 10th magnitude maximum observed in the middle of September, followed one fainter than 11 in the middle of March 1958.

205*27 RR Cap.* A very bright 8.5 magnitude maximum was observed in the middle of October.

214058 *μ Cep.* This star has been nearly constant for some years. In July 1958 it rose from a mean magnitude of about 4.5 to 4, and in March 1959 it became a little brighter than 4. It then decreased to about 4.5 in August, where it remained to the end of the year. The star is very red, and observations scatter almost one magnitude.

221255 *CP Lac* (*Nova 1936*). Fluctuated around 15th magnitude.

221*948 S Gru.* A very bright maximum, 6.5, was observed the first of the year.

225542 *SZ And.* A bright 9.5 magnitude maximum was observed early in June, following a 12th magnitude one in June 1958.

232848 *Z And.* A light curve of Z And was published in these Notes (vol. 48, no. 6, p. 222) in 1954, and it was suggested that we might look for another 8th magnitude maximum in a few years. In June 1959 the star rose rapidly from 11 to about 9½, but it levelled off and stayed fairly constant to the end of the year.

NOTE—ERRATA. In the light curve of Z And referred to above, the co-ordinates of figure 2 are given incorrectly. For magnitudes 8, 10 and 12, read 7, 9, and 11.

NOTE ADDED IN PROOF: R. Coronae Borealis began a drop to deep minimum in February 1960. During January it had fluctuated between 6 and 7, and according to Leif Robinson, it started down by February 11. On February 17, David Rosebrugh observed it fainter than 10, the faintest it has been since January 1953.

A.A.V.S.O. Nova Search Report (From George Diedrich, Chairman): Observations of Nova Search Areas were made by the following 25 observers for a total of 559 area-nights reported. Each name is followed by the number of observations made in September and then October, 1959: R. Bishop—0, 14; F. J. DeKinder—23, 24; D. Diedrich—7, 5; G. Diedrich—9, 7; C. L. Drolet—20, 11; P. Evans—45, 6; K. Fuller—41, 20; G. Gaherty, Jr.—10, 7; D. R. Humphrey—4, 0; H. C. Nightingale—3, 20; W. L. Isherwood, Jr.—25, 18; D. W. Orchiston—18, 19; C. G. Phillips—24, 30; R. Prezament—0, 2; R. T. Price—3, 3; D. Sands—0, 1; G. W. Smith—6, 1; G. A. Spencer—6, 7; A. Talevi—0, 22; F. Traynor—12, 15; W. A. Warren—0, 8; G. Wedge—16, 5; I. K. Williamson—6, 10; R. Zit—4, 1; K. Zorgo—14, 7.

Not previously reported are observations by the following two observers: R. Zit reports for May, June, July, and August totals of 4, 3, 7 and 1, respectively; K. Zorgo reports 22 observations for August.

Observations received during November and December 1959: A total of 10,248 observations was received for the two months; 4,900 from 65 observers in November, and 5,348 from 75 observers in December.

Observer	Nov.		Dec.		Observer	Nov.		Dec.	
	Var.	Ests.	Var.	Ests.		Var.	Ests.	Var.	Ests.
*Adams, R. M.	107	252	95	205	Lacchini, G. B.	50	230	29	77
Ahnert, P.	16	448	..	..	*Lattey, M.	7	23	13	13
Allin, R.	..	..	10	10	Ledbetter, N.	17	19	..	..
Anderson, C. E.	83	104	64	104	Lovi, G.	..	..	5	5
Anderson, E.	7	7	4	10	Lowder, W. M.	..	..	32	49
Balassa, T.	..	..	10	12	Martinez, A. E.	..	..	3	60
Ball, A. R.	6	6	..	..	McFarland, T.	1	1	..	..
Barrera, S.	..	..	1	1	McPherson, C. A.	19	19	13	13
Beidler, H. B.	17	18	10	10	Miller, W. A.	8	12	7	7
Berg, J.	24	76	..	..	Montague, A. C.	23	30	25	31
Borquez, P.	..	..	4	4	Morgan, F. P.	14	35	21	40
Brady, R. F.	10	10	24	53	Mourao, R. R.	5	5	..	..
Braune, W.	25	156	..	..	Muniz, L.	14	29	6	7
Breckinridge, J.	5	5	..	..	Nightingale, H. C.	..	..	1	1
Buckstaff, R. N.	8	21	..	..	Oravec, E. G.	173	387	224	459
Carbacho, P.	..	..	3	3	Orchiston, D. W.	..	..	66	370
Carlisle, J. H.	3	6	..	..	Ormazabal, M. E.	..	..	10	15
Celis, S. L.	..	..	12	43	Pearcy, R. E.	2	2	2	2
Cragg, T. A.	458	481	405	555	Peltier, L. C.	25	97	25	101
Darsenius, G.	9	41	13	21	Price, R. T.	..	..	16	50
de Kock, R. P.	135	551	134	526	Quester, W.	7	21	..	..
de la Vega, R.	..	..	68	506	Rettig, P.	..	..	8	13
de la Vega, X.	..	..	11	17	Rizzo, P. V.	14	18	20	30
Diedrich, DeL.	3	5	..	..	Robinson, J. C.	7	7	29	37
Diedrich, G.	2	3	3	4	Robinson, L. J.	1	1	3	4
Dudley, R. R.	..	..	4	6	Rosebrugh, D. W.	19	90	19	70
*Engelkemeir, D.	10	30	13	26	Royer, R.	6	6	..	..
Erpenstein, O. M.	12	22	11	27	Rudolph, R.	24	83	..	..
Evans, P.	..	..	21	164	Severin, M. A.	..	..	11	36
Farmer, R.	..	..	6	9	Sharpless, A. P.	..	..	8	8
Fernald, C. F.	243	550	179	293	Shinkfield, R. C.	..	..	19	41
Fletcher, D. M.	..	..	1	1	Smith, G. W.	4	4	..	..
Ford, C. B.	79	84	7	7	Smith, J. R.	29	32	5	15
Frogel, J. A.	..	..	4	4	Smits, P.	26	44	39	98
Fuller, K.	21	22	50	50	Solar, E.	..	..	2	6
Garland, G.	..	..	4	5	Solomon, L.	4	4	4	4
Godfrey, N. B.	35	36	..	..	Staer, R. R.	..	..	1	14
Goodsell, J. G.	3	3	..	..	Stanton, R.	9	9	30	39
Halbach, E. A.	..	..	2	2	Taboada, D.	..	..	152	206
Halvorson, D. O.	6	6	..	..	Templeton, J. H.	5	5	3	3
Hartmann, F.	154	173	161	172	Teran, M.	..	..	4	7
Hiett, L.	6	13	..	..	Traynor, F.	..	..	7	36
Hull, A.	2	2	..	..	Tsai, C. H.	50	127	..	..
Humphrey, D. R.	..	..	8	27	Valdez, M.	..	..	6	6
*Hutchings, N. O.	7	7	..	..	Van Til, A.	3	3	..	..
Judd, D.	10	14	4	4	van Zyl, L. L.	52	100	36	76
Kaminski, W. J.	11	11	19	35	Venter, S. C.	25	38	..	..
Kelly, F. J.	14	36	13	24	Williamson, L. J.	..	..	3	8
*Kindt, O. H.	17	20	13	18	Yamada, T.	56	175	61	317
Knowles, J. H.	21	25	12	16					

*In addition to their regular estimates, the following observers made continuous runs on the special variable V Sge: R. M. Adams, 1^h; D. Engelkemeir, 15^m, 15^m PEP; N. O. Hutchings, $1^h\ 15^m$; O. H. Kindt, 15^m; M. Lattey, 40^m.

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